## **Mathematical Modeling and Analysis**



# Quality Improvement of 3D Meshes in ALE simulation of Gas Dynamics

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Arbitrary Lagrangian Eulerian methods (ALE) methods have become an important tool in the simulation of gas dynamics problems since they incorporate the best features of both Lagrangian and Eulerian methods. In ALE methods, a Lagrangian step is first performed where the flow is calculated and the mesh is deformed according to the fluid flow. At the end of a Lagrangian step, a mesh optimization or rezoning step is performed to improve the quality of the mesh. The solution is then transferred from the Lagrangian mesh to the improved mesh and the simulation continued.

In this research, a procedure has been developed for the optimization of 3D mesh quality by node repositioning after the completion of the Lagrangian step. The procedure, called *Reference Jacobian based Mesh Optimization*, is designed to improve the quality or geometric shape of mesh regions and boundary mesh faces while keeping the improved mesh as close as possible to the original mesh. The method has been found to improve the quality of elements on external boundaries, material interfaces and in the interior while preserving mesh features and surface characteristics.

The Reference Jacobian based Mesh Optimization method for improving mesh quality consists of two stages. The first stage is a local optimization in which the optimal (reference) position of each mesh node is calculated with respect to the fixed positions of its neighboring nodes. The local objective function is based on the Jacobian matrix condition numbers of mesh elements connected to the particular node [1]. The reference positions of nodes are used to calculate two reference edge vectors for each edge in the mesh; each reference edge vector goes from the refer-

ence position of one node of the edge to the original position of the other. The reference edge vectors are then used to compute *Reference Jacobian Matrices* in the same way that Jacobian matrices were defined for the original mesh. The reference Jacobian matrices are used to construct a global objective function for the second stage of the procedure. The definition of this objective function directs the optimization to find a configuration for all the mesh edges such that a compromise is struck between the various pairs of reference edge vectors, the mesh remains valid and the element quality is improved.

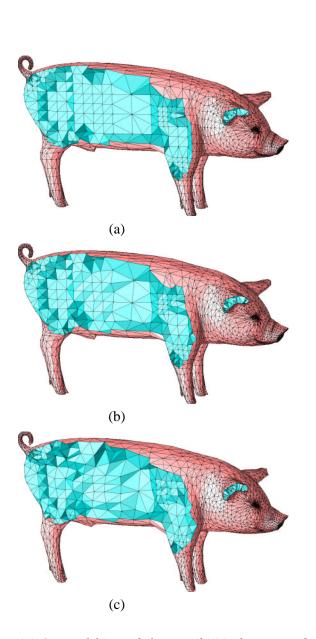
In a 3D mesh, interior nodes may be repositioned directly by optimizing an objective function with respect to its Cartesian coordinates. However, boundary nodes must be repositioned so that they remain on the original surface definition, and thereby maintain the essential characteristics of the surface. In this research, a novel technique has been devised to reposition boundary nodes so that they remain on the original discrete surface. Each node is moved in a local parametric space constructed by a barycentric or isoparametric mapping of the original mesh element it is moving in. If the node goes out of bounds in the local parametric space of one element, the procedure moves the node to the local parametric space of an adjacent element. Use of such local parametric spaces allows the procedure to be independent of an underlying smooth surface for the surface mesh and avoids the construction of a global parametric space which can be expensive.

The procedure has been tested on a number of surface and solid meshes and has proved to be very effective in improving mesh quality while minimizing changes to mesh features and surface characteristics.

#### References

[1] M. J. SHASHKOV AND P. M. KNUPP. Optimization-based reference-matrix rezone strategies for Arbitrary Lagrangian-Eulerian methods on unstructured grids. In *Proceedings of the Tenth Anniversary International Meshing Roundtable*, pages 167–176, Newport Beach, CA, October 2001. Sandia National Laboratories. Sandia Report SAND 2001-2976C.

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(a) Original 3D mesh for pig, (b) Mesh optimized with Reference Jacobian based objective function, (c) Mesh optimized with condition number objective function. Pictures show parts of the surface and interior meshes.

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